

QUANTITATIVE ANALYSIS – BLOCK REVISION MOCK 10

QUANTITATIVE TECHNIQUES

QUESTION ONE

- (a) Beta Glassworks Ltd. manufactures bottles for the beverage industry. The company specializes in producing bottles of three different capacities: 0.2 litre, 0.3 litre and 0.5 litre. The three types of bottles are made by two machines A and B. The number of hours required for each type of bottle on each machine is as shown below:

	Type of bottle		
	0.2 litre	0.3 litre	0.5 litre
Machine A	0.45	0.75	0.60
Machine B	0.30	0.15	0.75

Machine A has 3.825 hours available and machine B has 2.025 hours available. The management has decided that 1,500 bottles of 0.2 litre must be produced.

Required:

Determine how many bottles of 0.3 litre and 0.5 litre should be produced in addition to the 1,500 bottles of 0.2 litre. (6 marks)

- (b) Distinguish between input-output analysis and Markov analysis. (2 marks)

- (c) A miniature economy has three industries: motor vehicle, electricity and steel. These industries are interdependent., such that, the output of one industry is the input of another. The following table shows the input ratios of each industry.

Output	Input		
	Motor vehicles	Electricity	Steel
Motor vehicles	0.17	0.25	0.25
Electricity	0.25	0.25	0.33
Steel	0.50	0.33	0.33

The Leontif inverse matrix is computed as below:

$$\left(\begin{array}{ccc} & & \end{array} \right)$$

	3.08	1.98	2.15	
$(I - A)^1$		2.64	3.41	2.70
	3.96	3.19	4.46	

Required:

- (i) Define the term “input ratio.” (2 marks)
- (ii) Interpret the input column and output row for motor vehicles using the input ratios table. (2 marks)
- (iii) Determine the primary inputs required by each industry, if the final demand is Sh. 216 million, Sh. 240 million and Sh. 360 million for motor vehicles, electricity and steel industries respectively. (6 marks)
- (iv) The assumptions made in making the analysis above. (2 marks)

(Total: 20 marks)

QUESTION TWO

Leisure Publishers Ltd. recently published 20 romantic novels by 20 different authors. Sales ranged from just over 5,000 copies for one novel to about 24,000 copies for another novel. Before publishing, each novel had been assessed by a reader who had given it a rating between 1 and 10. The managing director suspects that the main influence on sales is the cover of the book. The illustrations on the front covers were drawn either by artist A or artist B. The short description on the back cover of the novel was written by either editor C or editor D.

A multiple regression analysis was done using the following variables:

- Y sales (millions of shillings).
- X₁ 1 if front cover is by artist A.
2 if front cover is by artist B.
- X₂ readers’ rating
- X₁ 1 if the short description of the novel is by editor C.
2 if the short description of the novel is by editor D.

The computer analysis produced the following results:

Correlation coefficient $r = 0.921265$

Standard error of estimate = 2.04485

Analysis of variance

	Degrees of freedom	Sum of squares	Mean square	F ratio
Regression	3	375.37	125.12	29.923
Residue	16	66.903	1.1814	

Individual analysis of variables:

Variable	Coefficient	Standard error	F Value
Constant	15.7588	2.54389	38.375
1	-6.25485	0.961897	42.284
2	0.0851136	0.298272	0.081428
3	5.86599	0.922233	40.457741

Correlation Coefficients

1	- 0.307729	0	- 0.674104
	1	0.123094	0.310838
		1	0.627329
			1

Required:

- The regression equation. (3 marks)
- Does the regression analysis provide useful information? Explain. (3 marks)
- Explain whether the covers were more important for sales than known quality of the novels. (4 marks)
- State with 95% confidence the difference in sales of a novel if its cover illustrations were done by artist B instead of artist A. (5 marks)
- State with 95% confidence the difference in sales of a novel if its short description was by editor D and not editor C. (5 marks)

(Total: 20 marks)

QUESTION THREE

- State the Baye's theorem (1 mark)
 - What is the importance of Baye's theorem to a business manager? (2 marks)

- (b) Excel Audit Consultants (EAC) have engaged three audit trainees; Odhiambo, Koech and Mwai. After three months, the audit supervisor notices that 3%, 5% and 4% of the accountants audited by Odhiambo, Koech and Mwai respectively have some procedural errors. The supervisor had distributed the accounts such that Odhiambo audited 40%. Koech audited 30% and Mwai audited 30% of the accounts.

Required:

- (i) The proportion of audited accounts with procedural errors. (6 marks)
- (ii) The probability that an account with an error is audited by either Koech or Mwai. (5 marks)
- (c) The finance manager of Apar Construction Ltd. has two investment proposals before him and he wishes to choose one of them. He will choose the project that offers him a higher expected net present value (NPV). In case both projects have the same NPV, he will prefer the project that has a lesser risk. The data for the two projects is given below:

Possible outcomes	Probability of possible outcomes	Net present value Sh. million
Project X:		
Pessimistic	0.25	1,500
Most likely	0.50	5,000
Optimistic	0.25	9,000
Project Y:		
Pessimistic	0.25	- 10,000
Most likely	0.50	5,000
Optimistic	0.25	20,500

Required:

The project the finance manager would choose, given that risk is measured by the standard deviation. (6 marks)

(Total: 20 marks)

QUESTION FOUR

- (a) Explain the difference between the paired t-test and the two-sample t-test. (4 marks)

(b) Trendy Tyres Ltd. has introduced a new brand of tyres, which in their advertisements claim to be superior to their only competitor brand, the Roadmaster Tyres. The brand manager of Roadmaster Tyres disputes this claim which he says is an advertisement gimmick. The brand manager of the two companies agree to run a road test for the brands. Ten (10) saloon cars of uniform weight and identical specifications are to be used for the test. Each car is fitted with both brands of tyres: One brand at the front and the other brand at the rear. The cars cover a distance of 5,000 kilometres and the tread wear is recorded as follows:

	Trendy tyres Centimetres	Roadmaster tyres Centimetres
1	1.08	1.12
2	1.06	1.09
3	1.24	1.16
4	1.20	1.24
5	1.17	1.23
6	1.21	1.25
7	1.18	1.20
8	1.10	1.15
9	1.22	1.19
10	1.60	1.13

Required:

- (i) Determine whether Trendy Tyres Ltd.'s claim is true using $\alpha = 0.01$. (15 marks)
- (ii) What are the assumptions you have made in (i) above? (1 mark)

(Total: 20 marks)

QUESTION FIVE

- (a) State the principal components of a time series. (2 marks)
- (b) (i) Explain the difference between multiplicative and additive models as used in time series. (2 marks)
- (ii) State the conditions under which each model is used. (2 marks)
- (c) The table below shows the sales of new cars by quarters during a period of three years:

Year	Quarter 1 Sh. "million"	Quarter 2 Sh. "million"	Quarter 3 Sh. "million"	Quarter 4 Sh. "million"
2001	55.0	76.5	61.2	77.8
2002	54.4	65.9	52.7	81.4
2003	59.3	83.2	78.5	93.0

Required:

- (i) Explain the purpose of the seasonal index (2 marks)
- (ii) The seasonal index for each quarter assuming an additive model. (12 marks)

(Total: 20 marks)

SECTION II

QUESTION SIX

- (a) A company manufactures mountain and racing bikes. Each mountain bike is sold at Sh. 6,375 and a racing bike is sold at Sh. 9,000. A racing bike requires 6 kilogrammes of aluminium while a mountain bike requires 4 kilogrammes of aluminium. Aluminium cost Sh. 750 per kilogramme. It costs the company Sh. 375 per hour to assemble each bike (irrespective of model). It takes 1 hour to assemble a racing bike and 2 hours to assemble a mountain bike. There are 4,800 kilogrammes of aluminium available and a total of 1,600 hours available for assembly.

Required:

- (i) Formulate the linear programming problem. (6 marks)
- (ii) Graphically solve the problem formulated in (i) above. (10 marks)
- (b) Is it appropriate to use non-graphical linear programming methods to solve the problem in (a) above? Why? (2 marks)
- (c) State some of the simplifying assumptions you have made to solve the problem in (a) above. (2 marks)

(Total: 20 marks)

QUESTION SEVEN

(a) State the limitation of using the expected monetary value criterion in decision making.

(2 marks)

(b) The Metro Transport Company (MTC) Ltd. managers are in the process of deciding how to service their vehicles. Currently, Spanner Motors Ltd. have been servicing the fleet at a fixed cost of Sh. 2.4 million per year. The company (MTC) has a fleet of 20 public service vehicles (PSV). The management may renew the contract with Spanner Motors Ltd. or choose from any of two other alternatives available.

Alternative A:

Auto Care Ltd. (AL) offers to charge an annual fixed cost of Sh. 2 million for the first 80 service checks and repairs. However, any service check and repair above the first 80 is charged at Sh. 12,000 per service. From past experience and the records available, the management of MTC Ltd. have the following information.

Service demand per annum	Probability	Maximum (annual) cost Sh. "million"
80 and below	0.6	2.00
81 – 100	0.3	2.24
100 - 150	0.1	2.60

SUGGESTED ANSWERS

QUESTION ONE

Let X = Number of 0.2 litre bottles

Y = Number of 0.3 litre bottles

Z = Number of 0.5 litre bottles

$$0.45x + 0.75y + 0.6Z = 3825$$

$$0.30x + 0.15y + 0.75Z = 2025$$

$$\text{but } X = 1500$$

$$0.75y + 0.6Z = 3150$$

$$0.15y + 0.75Z = 1572 \times 5$$

$$0.75y + 3.75Z = 7875$$

$$\underline{0.75y + 0.60Z = 3150}$$

$$3.15Z = 4725$$

$$Z = 1500 \text{ bottles}$$

$$0.75y + 0.06(1500) = 3150$$

$$y = 3000 \text{ bottles}$$

- (b) Difference between input-output analysis and Markov analysis – input – output analysis shows the interdependence of sectors in an economy.

Forecasted levels of output required for each sector so as to satisfy both intermediate and final demand can be calculated if we are given the technical coefficient matrix and the forecasted levels of final demand. $X = (1 - A)^{-1}D$

Markov analysis is a probabilistic system whereby the state of a given phenomenon in future can be predicted from the current state and transition matrix (initial state vector)(Transition matrix) = (Future state vector)

- (c) (i) Input ratio is the proportion of inputs an industry receives from another industry or from itself.
- (ii) Interpretation of the column of motor vehicles:
- Motor vehicles receive 17% of its inputs from itself.
 - Motor vehicles receive 25% of its inputs from electricity.
 - Motor vehicles receives 50% of its inputs from steel.
- Interpretation of the row for motor vehicles.
- + 17% of motor vehicle output is distributed to itself.
 - + 25% of motor vehicle output is distributed to electricity.
 - + 25% of motor vehicle output is distributed to steel.

$$(iii) \quad \text{Total output} = (1 - A)^{-1}X$$

$$= \begin{bmatrix} 3.08 & 1.98 & 2.15 \\ 2.64 & 3.41 & 2.70 \\ 3.96 & 3.19 & 4.46 \end{bmatrix} \begin{bmatrix} 216 \\ 240 \\ 360 \end{bmatrix} = \begin{bmatrix} 1914.48 \\ 2360.64 \\ 3226.50 \end{bmatrix}$$

Proportion of primary inputs are:

$$\text{Motor vehicles} = 1 - (0.17 + 0.25 + 0.5) = 0.08$$

$$\text{Electricity} = 1 - (0.25 + 0.25 + 0.33) = 0.17$$

$$\text{Steel} = 1 - (0.25 + 0.33 + 0.33) = 0.09$$

Therefore primary inputs required are:

$$\text{Motor vehicles} = 0.08 \times 1914.48 = \text{Shs. } 153.1584 \text{ million}$$

$$\text{Electricity} = 0.17 \times 2360.64 = \text{Shs. } 401.3088 \text{ million}$$

$$\text{Steel} = 0.09 \times 3226.56 = \text{Shs. } 290.3904 \text{ million}$$

(iv) Assumptions of input/output analysis:

- Each industry produce a single homogenous product.
- The output of each industry is subject to constant return.
- Each industry requires a fixed input ratio to meet its demand.
- No new industries are allowed.

QUESTION TWO

(a) Regression equation, $\hat{y} = 15.7588 - 6.25485X_1 + 0.0851136X_2 + 5.86599X_3$

(b) The regression analysis provide useful information

$$r = 0.921265 \Rightarrow R^2 = 0.85$$

- The regression equation explains about 85% of the variation in sales.
- There is a high positive linear relationship between the sales and the independent variables.

(c) Generally the bigger the value of F, the better the predictor readers rating have a small value of F compared with the other decision variables. Curves have a bigger F value. This implies that curves are important for sales prediction.

(d) $\alpha = 5\%$
 $\frac{\alpha}{2} = 0.025$
 $dif = 20 - 4 = 16$

} t - Critical = 2.12

$$\begin{aligned}
 95\% \text{ confidence interval} &= -6.25485 \pm 2.12 \times 0.961897 \\
 &= -6.25485 \pm 2.03922 \\
 &= -8.29407 < B_1 < -4.21563
 \end{aligned}$$

We are 95% confident that a cover by artist B instead of artist A reduces sales by between 4216 and 8294 copies.

(e) $95\% \text{ confidence interval} = 5.86599 \pm 2112 \times 0.922233$
 $= 5.86599 \pm 1.95513$
 $= 3.911 < B_3 < 7.821$

We are 95% confident that using short descriptions by Editor D instead of Editor C will increase sales by 3911 and 7821 copies.

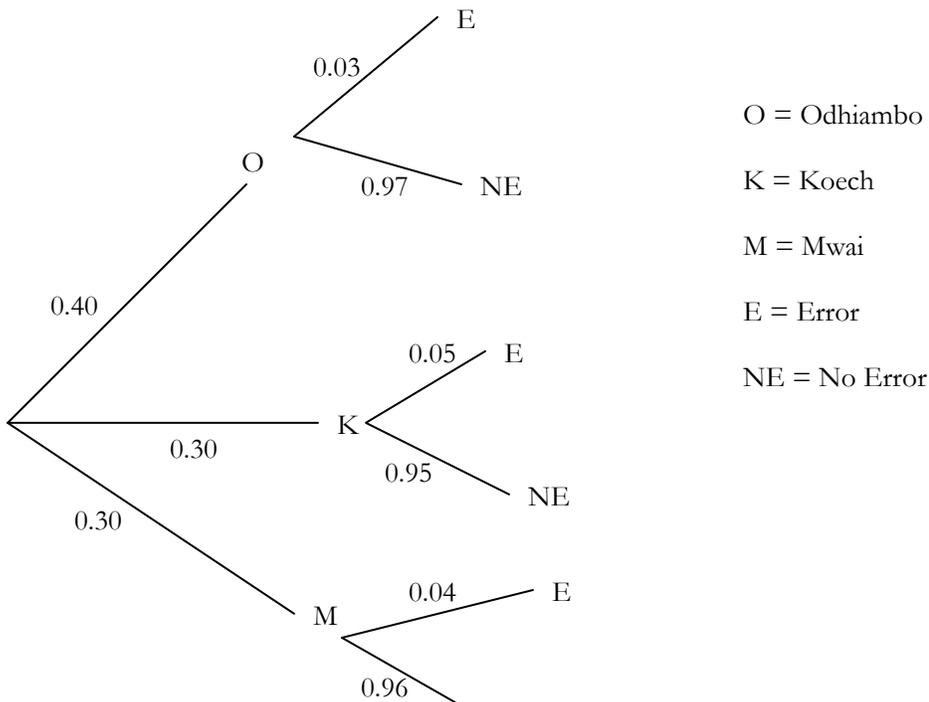
QUESTION THREE

- (a) (i) General form of Bayes theorem:
 If A and B are related events then:

$$P(B|A) = \frac{P(A|B) \times P(B)}{P(A)}$$

$$P(A) = P(A|B_1)P(B_1) + P(A|B_2)P(B_2) + \dots + P(A|B_N)P(B_N)$$

- (ii) Importance of Bayes theorem to a business manager.
- Its used in revision of probabilities.
 - Business events keep on changing
 - Bayes theorem helps managers to update their projections using available additional information.



$$P(E) = (0.4 \times 0.03) + (0.3 \times 0.05) + (0.3 \times 0.04) = 0.039 = 3.9\%$$

$$\begin{aligned} \text{(iii)} \quad (K \text{ or } M|E) &= P(K|E) + P(M|E) \\ &= \frac{P(K \& E)}{P(E)} + \frac{P(M \& E)}{P(E)} \\ &= \frac{0.3 \times 0.05}{0.039} + \frac{0.3 \times 0.04}{0.039} \\ &= 0.3846 + 0.3077 \\ &= 0.6923 \end{aligned}$$

(c)

Project X				Project Y			
X_1	P_1	P_1X_1	$(X_1 - \bar{X})^2 P_i$	y_i	P_i	$P_1 y_1$	$(Y_1 - \bar{Y})^2 P_i$
1500	0.25	375	3,285,156.25	-	0.25	-	57,191,406.25
				10,000		2,500	
5000	0.50	2500	7,812.50	5,000	0.50	2500	7,812.50
9000	0.25	2250	3,753,906.25	20,500	0.25	5125	59,097,626.25
	$\bar{X} =$	5125	$6x^2 =$		$\bar{Y} =$	5,125	$6y^2 = 116,296,875$
			7,046,875				

$6x =$ Shs. 2,655 million

$6y =$ Shs. 10,784 million

Since the two projects have the same expected net present value ($\bar{X} = \bar{Y}$) we choose project x because $6x < 6y$.

QUESTION FOUR

- (a) Difference between paired t-test and two sample t-test.
- Paired t-test is used for the mean of differences where samples are not independent.
 - Two-sample t-test is used to test for the difference in means where samples are independent.

Trend wear of trendy tyres	Trend wear of Road tyres	master (d)	Differences (d - \bar{d}) ² = (d + 0.021) ²
1.08	1.12	- 0.04	0.000361
1.06	1.09	- 0.03	0.000081
1.24	1.16	0.08	0.010201
1.20	1.24	- 0.04	0.000361
1.17	1.23	- 0.06	0.001521
1.21	1.25	- 0.04	0.000361
1.18	1.20	- 0.02	0.000001
1.10	1.15	- 0.05	0.000841
1.22	1.19	0.03	0.002601
1.09	1.13	- 0.04	0.000361
	Σ	- 0.21	

$$\text{Mean: } \bar{d} = \frac{\Sigma d}{n} = \frac{-0.21}{10} = -0.021$$

Standard deviation $S = \sqrt{\frac{\sum (d - \bar{d})^2}{n-1}} = \sqrt{\frac{0.01669}{9}} = 0.0431$

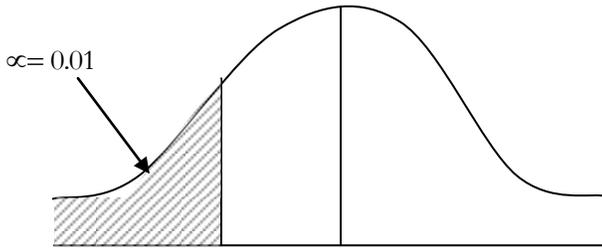
$H_0: d = 0$ (Trendy tyres not superior or the two are equally superior)

$H_1: d < 0$ (Trendy tyres are superior)

Test statistics, t – calculated

$$= \frac{\bar{d}}{S/\sqrt{n}} = \frac{-0.021 \times \sqrt{10}}{0.0431} = -1.54$$

$$\left. \begin{array}{l} \alpha = 0.01 \\ \text{d.f.} = 10 - 9 = 9 \end{array} \right\} t - \text{Critical} = t_{0.99} = 2.82$$



$$T = -2.82$$

Decision Rule: Reject H_0 if t – calculated < -2.82

Since $-1.54 > -2.82$, we fail to reject H_0 and conclude that trendy tyres are not superior.

- (ii) Assumptions made
- Trend wear is normally distributed.

QUESTION FIVE

- (a) Principal components of a time series are:
- Secular trend (T)
 - Seasonal variation (S)
 - Cyclic variation (C)
 - Random variation (R)
- (b) (i) Difference between multiplicative and additive models:

- Multiplicative model expresses the time series model as a product of the four principle components.

That is $Y = TSCR$

- Additive model expresses the time series model as a sum of the four principle components.

That is $Y = T + C + R + S$

(ii) Conditions under which each model is used;

- Multiplicative model is used if the four principle components are not independent.
- Additive model is used when the four principle components are independent.

(c) (i) Purpose of seasonal index

- Used to evaluate seasonal effects on a time series.
- Used to adjust trend forecasts.
- Used to deseasonalise data.

(ii)

Year	Quarter (Q)	Sales (Y)	Uncentred 4 – Quarter MA	Centred 4 – Quarter M.A	Difference Y - MA
2001	1	55.0	-	-	-
	2	76.5	67.625	-	-
	3	61.2	67.475	67.55	- 6.36
	4	77.8	64.825	66.15	11.65
2002	1	54.4	62.700	63.76	- 9.36
	2	65.9	63.600	63.15	2.75
	3	52.7	64.825	64.21	- 11.51
	4	81.4	69.150	66.99	14.41
2003	1	59.3	75.600	72.38	- 13.08
	2	83.2	78.500	77.05	6.15
	3	78.5	-	-	-
	4	92.0	-	-	-

Subsidiary Table

Year	Quarter 1	Quarter 2	Quarter 3	Quarter 4
2001	-	-	- 6.35	11.65
2002	- 9.36	2.75	- 11.51	14.41
2003	- 13.08	6.15	-	-
Total	- 22.44	8.90	- 17.86	26.06
Average Seasonal Index	- 11.22	4.45	- 8.93	13.03

QUESTION SIX

- (a) Let: X = Number of mountain bikes
Y = Number of racing bikes
P = Total contribution

Unit contribution of mountain bikes = $6,375 - (4 \times 750 + 2 \times 375) = \text{Shs. } 2,625$

Unit contribution of racing bikes = $9,000 - (6 \times 750 + 1 \times 375) = \text{Shs. } 4,125$

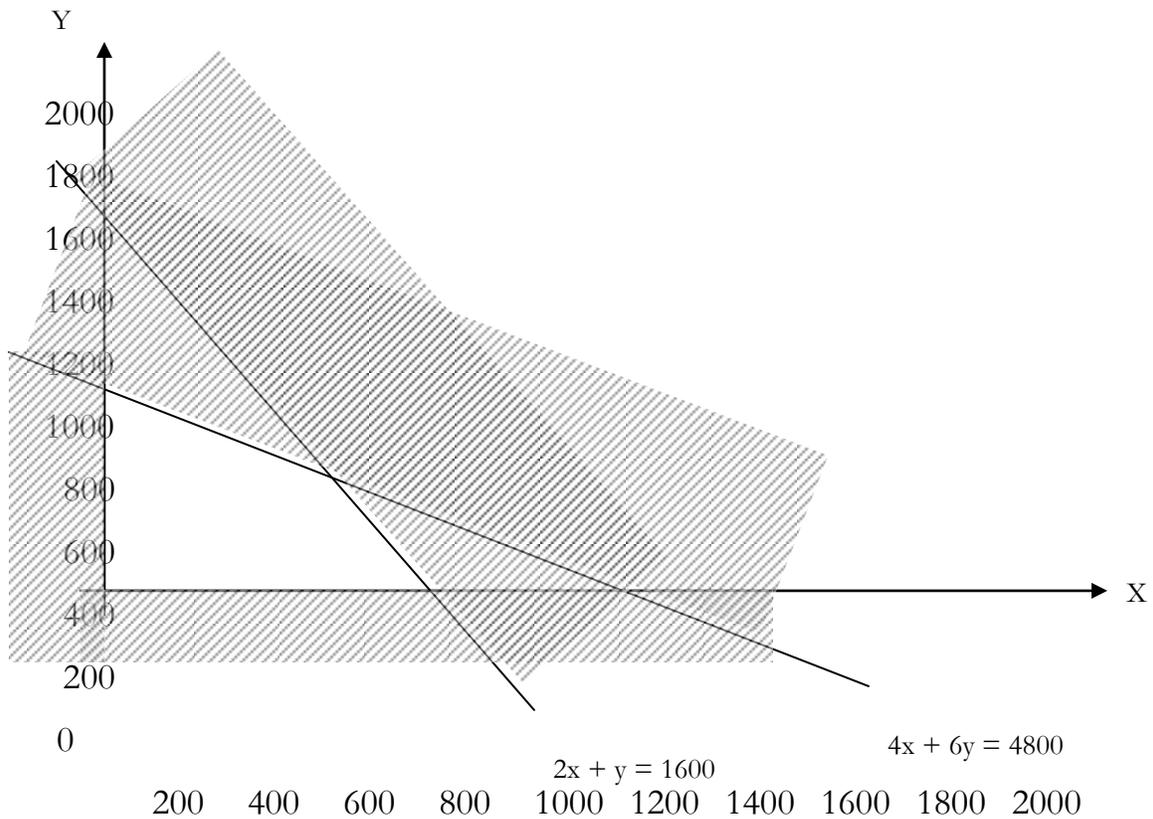
Max. $P = 2,625x + 4,125y$

Subject to: 1. $4x + 6y \leq 4,800$

2. $2x + y \leq 1,600$

3. $x, y \geq 0$

X	0	1200
Y	800	0
X	0	800
X	1600	0



Conner Point (X, Y)	$P = 2625x + 4125y$
A (0, 800)	$P = \text{Shs. } 3,300,000$
B (600, 400)	$P = \text{Shs. } 3,225,000$
C (800, 0)	$P = \text{Shs. } 2,100,000$

Optimal solution

$X = 0$ mountain bikes $Y = 800$ Racing bikes
 Max. $P = \text{Shs. } 3,300,000$

(b) Yes. Simplex method can be used

(c) Assumptions of linear programming

- All functions are linear and can be stated mathematically
- Coefficients of the decision variables are known with certainty.
- Resources and decision variables can be added linearly
- Decision variables are divisible
- Proportionality
- There is only one objective

- All decision variables are positive.

QUESTION SEVEN

- (a) The limitation of using the expected monetary value criterion in decision making is that it ignores the variation in risk among alternative decisions.

Project A

X_1	P_1	$P_1 X_1$	$(X_1 - \bar{X})^2 P_1$
2.00	0.6	1.200	0.0104544
2.24	0.3	0.672	0.0034992
2.60	0.1	0.260	0.0219024
	$\bar{X}_A =$	2.132	$\delta_A^2 = 0.035856$

Project B

y_1	P_1	$P_1 y_1$	$(y_1 - \bar{y})^2 P_1$
1.8	0.20	0.36	0.052020
2.4	0.75	1.80	0.006075
3.0	0.05	0.15	0.023805
	$\bar{Y}_B =$	2.31	$\delta_B^2 = 0.0819$

- (i) Expected cost of alternative A $\bar{X}_A =$ Shs. 2.132 million
 Expected cost of alternative B $\bar{Y}_B =$ Shs. 2310 million
 Cost of mechanics = Shs. 25,000 \times 5 \times 12 = Shs. 1.5 million
 Total cost of alternative B = 2.31 + 1.5 = Shs. 3.81 million

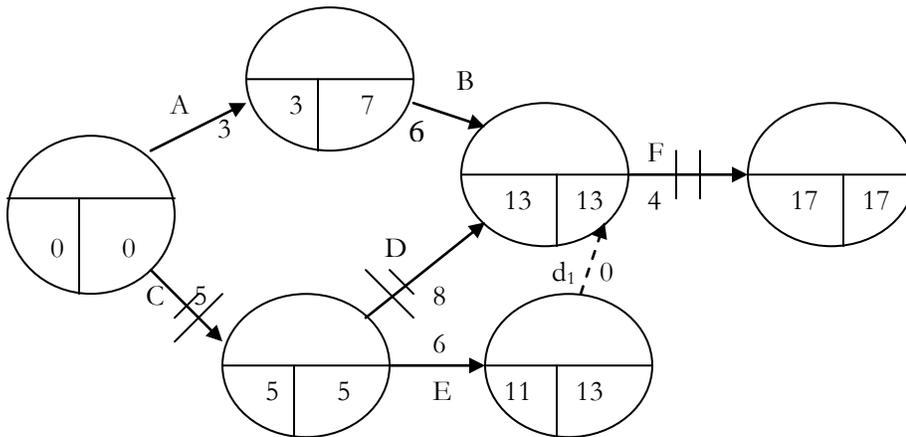
Recommendation

Adopt alternative A since it is the least expensive.

- (ii) $\bar{X} = 2.132$ $\bar{Y}_B = 2.31$
 $\delta_A = 0.1894$ $\delta_B = 0.2862$
 $C.V_A = \frac{0.1894}{2.132} \times 100 = 8.88\%$ $C.V_B = \frac{0.2862}{2.31} \times 100 = 12.3\%$

- (iii) Alternative A is better because it has a lower risk.

QUESTION EIGHT



Normal completion time = 17 weeks
 Critical activities are C, D and F

(b) (i)

Activity	Time Reduction	Cost (Shs.)	Slope
A	0	-	
B	2	45,000	
C	2	30,000	
D	1	60,000	
E	2	22,500	
F	2	75,000	

Path	Time	Crash C by 2	Crash D by 1	Crash F by 2
G, D, F	17	15	14	12
A, B, F	13	13	13	11
G, E, F	15	13	13	11
Additional		2 x 30 = 60	60 + 60 x 1 = 120	120 + 75 x 2 = 270

Cost Shs. '000'

Shortest

(ii) Additional cost if the project is crashed = Shs. 270,000

(c) Cost slope is the additional cost that is incurred when an activity time is reduced by one unit.

$$\text{Cost slope} = \frac{\text{Crash cost} - \text{Normal cost}}{\text{Normal time} - \text{Crash time}}$$

(d) Assumptions made when crashing

- Cost slope is constant
- There is a direct relationship between time and costs